Geostatistical analysis of temperature in the boundary layer using variogram scaling exponents

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Abstract

Variogram analysis is commonly used in the geospatial sciences to quantify spatial autocorrelation in both human and physical phenomena. However, the use of geostatistics and variograms has rarely been extended for vertical atmospheric measurements, where geographic principles such as spatial autocorrelation may play a key role in understanding turbulence. Turbulence remains one of the most unpredictable and least-understood geographic phenomena. Small-scale temperature fluctuations are important for understanding turbulent motions, but until recently, sampling these fluctuations throughout the vertical extent of the boundary layer was difficult. Geospatial technologies such as unmanned aircraft systems (UAS) are allowing researchers to capture data at the spatial and temporal resolutions necessary to test long-standing theories, and they are also facilitating the use of spatial analytical research. Quantifying the structure of turbulent features has been challenging, but variograms may offer a solution. Building on existing research, this study extends variogram analysis into the lower atmospheric boundary layer (ABL) to determine whether the scaling exponents in power model variograms fit to data captured from a UAS contain information relevant for understanding turbulence. Results indicate that model errors are more correlated during times when turbulence is supressed, leading to scaling exponents above 1. As turbulence increases from both radiative and mechanical sources, scaling exponents trended toward 1 and 0, indicating more random correlation, or even negative correlation of the model errors. While additional work is needed across a variety of environments and conditions, these findings have the potential to aid meteorologists and weather forecasters in better understanding turbulence.

Keywords: drones; geostatistics; scale and scaling; unmanned aerial vehicles; atmospheric physics; planetary boundary layer.

1 Introduction

Small-scale temperature fluctuations are important for meteorology because the small-scale, turbulent motions that cause these fluctuations are likely to be universal (compared to larger-scale motions), which could lead to a better understanding of turbulence at all scales (Sreenivasan and Antonia 1997). Many decades ago, Kolmogorov (1941) proposed that within a certain range of the atmosphere, termed the 'inertial subrange', energy containing eddies are homogenous (spatially stationary), isotropic, and independent of the larger, energy containing eddies in which they are nested. These properties adhere to several laws in Geographic Information Science (according to Goodchild 2004) including Tobler's First Law and the so-called Second Law, or principle, of heterogeneity (or statistical nonstationarity), which posits that there is no 'average' place on earth and that geographic variables exhibit uncontrolled variance. Given these similarities between atmospheric phenomena and geographic phenomena, it is not surprising that the statistic commonly employed by atmospheric scientists to characterize eddies in the inertial range, known as the temperature structure function (Eq. 1), closely resembles the variogram, which is a cornerstone of geostatistics. The temperature structure function is:

$$\Delta \mathrm{T}_h^2 = [T(x+h) - T(x)]^2 \tag{1}$$

where T is the temperature measured at location x, and h is the separation distance between two measurements. In comparison, the variogram is calculated as (Oliver and Webster 2015):

$$2\gamma h = E[\{Z(x) - Z(x+h)\}^2]. \tag{2}$$

Thus, the variogram used in geostatistics is analogous to the temperature structure function used in atmospheric science. However, beyond the structural similarities of the equation, variogram analysis has not been applied extensively in meteorology and atmospheric physics (but see Sheuerer and Hamill 2015). Thus, there is room to advance understanding of atmospheric phenomena, particularly turbulence, through geostatistical analyses.

In particular, in the inertial subrange, the temperature structure parameter has been found to obey the relation:

$$\Delta T_h^2 = {C_T}^2 h^{\frac{2}{3}} \tag{3}$$

where C_T^2 is the temperature structure parameter and acts as a proportionality factor (Wyngaard et al. 1971). Eq. 3 is analogous to a power law function:

$$y = ax^k \tag{4}$$

where a is a constant, and k is the scaling exponent. When the variable x in Eq. 4 is scaled by a constant factor (in the case of the temperature structure function, ${C_T}^2$), the function itself is scaled proportionately. In other words, power law functions are scale invariant, which means they are an attractive model for studying geospatial phenomena where scale and scaling confound analyses (i.e., the modifiable areal unit problem, or MAUP). Since atmospheric processes are often intrinsic but not second-order stationary, power law functions are well-suited for modelling.

In theory, Eq. 3 can be solved to determine the separation distances at which the temperature structure parameter is valid for scaling measurements, but until recently, acquiring the fine-scale data needed to test these scaling relationships has been difficult. Historically, atmospheric sampling has relied almost entirely on (1) meteorological towers, which are fixed in space and have limited vertical reach (~10m), and (2) weather balloons, which are launched infrequently both in space and time. Weather balloons can sample to a higher altitude compared to towers, but their path is uncontrolled, and the spatial and temporal resolution of the data is often not fine enough to capture the small-scale measurements of interest. To fill these gaps, geospatial technologies such as unmanned aircraft systems (UAS, or drones) are being been adopted by atmospheric scientists at unprecedented rates because they can sample portions of the atmosphere that are beyond the reach of meteorological towers, their position in space can be controlled, and samples can be collected at systematic spatial and temporal scales (Hemingway et al. 2017).

A recent study by the authors used UAS-acquired temperature measurements and variogram analysis to determine the sample separation distances and above ground altitude of the inertial subrange by assessing the agreement between the sample variogram and a power law variogram model. In that study, the power law variogram was forced to fit with a 2/3 exponent since the scaling exponent in Eq. 3 is 2/3. However, the authors found that in some cases, the sample variogram deviates considerably from the 2/3 power law for certain separation distance ranges. While the portions of the sample variogram that correspond to the theoretical 2/3 model variogram are hypothesized to be the inertial subrange, portions outside that limited range may hold key information for understanding atmospheric phenomena. These deviations may relate to turbulence, but it is not yet understood why they occur (e.g., altitudes, weather conditions, surface environmental factors, etc.), where they occur within the atmosphere, or whether their occurrence and position might be predictable, which could lead to better use of variograms for capturing the structure of atmospheric phenomena and ultimately a better understanding of atmospheric turbulence.

The objective of this study is to explore the structure of the sample variogram across a set of vertical profiles of temperature to determine how the scaling exponent from a power model variogram fit with least squares may provide additional information related to meteorological conditions. Specifically, this study (1) develops sample variograms for six, vertical profiles of temperature captured during UAS flights that took place on 18 July 2018 in Colorado, USA, (2) fits power model variograms to the data using a least squares approach, and (3) compares the scaling exponents from the variograms fit to both the original data as well as detrended data to uncover whether inferences can be drawn on the magnitude of the exponent based on surface conditions.

2 Materials and Data Collection

Temperature measurements were collected on 18 July 2018 in the San Luis Valley of Colorado, which is located in the south central region of the United States. Data were collected using a DJI M600 (DJI Shenzhen, China), which is a hexacopter UAS measuring 1.13 m in diameter and weighing 9.1 kg. The payload sensor was a Young Model 81000 ultrasonic anemomenter (R.M. Young Company, Minnesota), which weighs 1.2 kg (Fig. 1). The anemometer measures temperature at a resolution of 0.01 m s^{-1} with an accuracy of ± 2° C at 30 m s^{-1}. Data were logged at a rate of 32 Hz.

Figure 1. DJI M600 platform at the sampling site with sonic anemometer attached on top (Photo: JJ)

Six flights were conducted on 18 July in Moffat, Colorado, which is located at 2320 m MSL. The site is dominated by grassland and some deciduous shrubs (Fig. 1). Flights spanned the morning boundary layer transition with the first flight taking place at 07:08 local time (UTC-6) and the last flight occurring at 12:56 local time (Table 1). Flights were conducted under a Certificate of Authorization (COA) issued by the U.S. Federal Aviation Administration, which allowed the maximum altitude to exceed the U.S. mandated maximum of 123 m. Flight formations were vertical profiles with data collected only during the ascent to ensure temporal stationarity.

Table 1. Flight information for the six profile flights

Flight	Start time	End time	Max altitude
1	07:08:02	07:10:48	490.9
2	07:59:41	08:02:42	499.2
3	08:59:43	09:02:45	499.2
4	10:59:56	11:01:48	499.7
5	11:46:50	11:49:47	498.2
6	12:30:49	12:32:32	309.8

Surface meteorological conditions were obtained from a weather station located at the San Luis Valley Regional Airport (NOAA), located approximately 60 km from the sampling site in Moffat, Colorado.

3 Methods and Calculations

Matheron's (1963) method-of-moments estimator was used to compute sample variograms from the six vertical profiles:

$$2\hat{\gamma}(h) = \frac{1}{N(h)}\sum_{i=1}^{n(h)}\{Z(x_i) - Z(x_i + h)\}^2 \qquad (3)$$

where $Z(x_i)$ is the observed value of Z at location x_i separated by distance h, and N is the number of sample pairs (Cressie, 1993). In the case of atmospheric profiles of temperature where the temperature varies with height, the mean is not stationary. This variation causes the intrinsic hypothesis to be invalid as well as causing the variogram to increase without bound (i.e., the variogram does not ever reach a sill). Thus, a mean function was applied to account for the variation in temperature with height. Furthermore, Stull (1988) acknowledges that by subtracting the mean from a random process, the turbulent part of the process can be isolated, and since improving understanding turbulence is one goal of this study, applying a mean function is warranted.

Next, power model variograms were fit to the set of sample variograms using the equation:

$$2\gamma h = wh^k \qquad (4)$$

where w is the intensity of the variation, and k is a scaling factor (Webster and Oliver 2007). The coefficient a is equal to the temperature structure parameter, C_T^2. The power model variogram is only valid when 0 > α > 2 (Chiles and Delfiner 2012). Lastly, the scaling exponents from the power variogram models were compared across the six flights and interpreted in the context of surface atmospheric conditions.

4 Results and Discussion

The temperature profiles from the six flights are typical of the evolution of the atmospheric boundary layer (the portion of the atmosphere in contact with the Earth) across the morning. The first three flights (Fig. 2) show evidence of a nocturnal inversion, where thermal stratification results from radiative cooling. When this situation occurs, temperatures are cooler near the surface of the Earth and increase with height. During these nocturnal inversions, turbulence is usually suppressed by the strong thermal stratification. The first three flights (Fig. 2) show this inversion occurring around 200-250 m AGL where there is a discontinuity in the upward trend of the profile. Throughout the morning, as the sun warms the surface of the Earth, the air becomes mixed, and the temperature inversion is no longer present in the final three flights (Fig. 2).

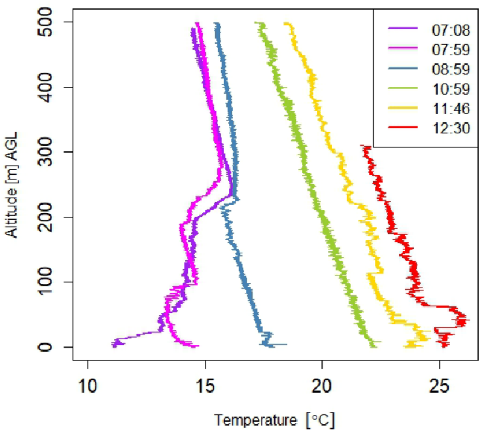

Figure 2. Temperature profiles captured on 18 July 2018

As radiative heating occurs, convective turbulence increases. Additionally, on the day of these flights, mechanical turbulence caused by wind also increased throughout the morning. Data from the NOAA station confirm that winds were generally calm until 11:52, at which point they became variable at 1.3 m s^{-1} with gusts measured at 7.6 m s^{-1}. By 12:52, approximately 20 minutes after the final flight of the day (red plot, Fig. 2), winds were sustained from the north at 6.3 m s^{-1} with gusts reaching 11.6 m s^{-1}. It should be noted that the final flight (red plot, Fig. 2) was aborted at 300 m AGL due to strong winds.

Scaling parameters (k) for the power model variograms fit to the sample variograms ranged from 0.13 to 1.17 (Fig. 3). In general, scaling exponents for the three early flights, which occurred when there was still a nocturnal inversion in place, have the highest scaling exponents (Fig. 3a-c), while the three profiles captured during the latter part of the day had much lower scaling exponents (Fig. 3d-f). The first and third flights (07:08 and 08:59) had scaling exponents above 1.0 (Fig. 3), and also have the most typically looking profiles for early morning flights when turbulence should be supressed. The 07:59 flight closely resembles the 08:59 flight above 100 m AGL, but below that height it has an anomalous bowed structure, which may have contributed to the lower scaling exponent ($k = 0.81$).

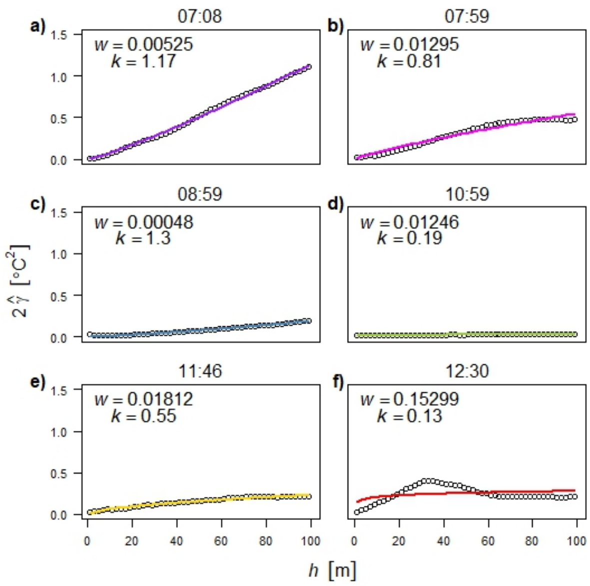

Figure 3. Sample (circles) and power model variograms (lines) with model coefficients (w) and scaling exponents (k)

According to Webster and Oliver (2007), when the scaling exponent is close to 2, the model errors are perfectly correlated, and there is differentiable variation in the underlying process. As the scaling exponent trends to 1, it signals a random walk model. As the scaling exponent trends below 1, the error terms become negatively correlated, and for exponents of 0, the signal is pure nugget. The latter three profiles all have scaling exponents below 1 (trending toward 0), which suggests that the signal is becoming pure nugget. In particular, the final profile of the day had a scaling exponent of 0.13, suggesting that there is negative spatial autocorrelation in the values at different separation distances. Consequently, this profile was also captured latest in the day (12:30) when radiative mixing should have been greatest as well as during the period of strongest winds. Both factors suggest that turbulence in this profile would have been greater compared to the other profiles.

While we did not expect any of the datasets to generate a power model variogram with a 2/3 exponent (the theoretical value for the inertial subrange), it is useful to note that the flight at 11:46 was close ($a = 0.55$). It can be seen both in the plot (yellow line, Fig. 2) and from observations recorded at the NOAA station that day indicating surface winds began to increase right about this time. These data suggest that sample variograms for profiles in which turbulence is moderate—either due to radiative heating or wind—may follow the expectation of a 2/3 exponent more so than profiles in which turbulence is suppressed or those in which mechanical forces (e.g., wind) are strong.

It is useful to note that with the power model variogram, it is possible to discern the smoothness/roughness of the field being measured. During the final flight, which was aborted at 300 m due to the presence of strong winds (Fig. 3f), there were large fluctuations (i.e., roughness) in the measured temperature values, which manifested in a sample variogram that did not increase smoothly with distance. In cases such as this, the power model variogram may not be particularly relevant for capturing spatial structure (or lack thereof). Indeed, the model fit to this profile produced a scaling exponent very close to 0, which suggests that the signal being captured is negatively correlated and almost pure noise.

The overall findings of this variogram analysis indicate that there may be certain conditions under which variogram analysis is appropriate for uncovering the structure of turbulence in the atmosphere and other conditions when this technique may not be suitable. The overall decrease in the scaling exponent across the day suggests that the model errors become less correlated as radiative mixing is occurring and as winds are increasing, but the relative contribution of these two forces to the scaling exponent are not yet known.

5 Conclusions

Using geostatistical analysis, specifically model variograms, this research investigated how the scaling exponent can potentially be used to understand the spatial structure of turbulence in the atmospheric boundary layer. Under stable conditions, which typically occur early in the morning when radiative cooling from the prior night creates a temperature inversion with little turbulence, the scaling exponents of the variogram were above 1, indicating more spatially correlated errors terms. As radiative heating increased throughout the morning and the lower atmosphere became mixed, scaling exponents decreased below 1. As mechanical forces (in the form of wind) increased during the final flights, the scaling exponent trended toward 0, indicating the signal is almost entirely noise. While much more work is needed to fully understand how variograms may uncover the spatial structure of atmospheric phenomena, these findings demonstrate how geostatistical analyses can be translated to vertical measurements of the boundary layer to explore spatial autocorrelation.

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