A GIS-based approach to karst relief cyclicity by using Fast Fourier transform

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Abstract

The aim of this paper is to use an automated approach to challenge the cyclicity of the bare Earth surface phenomena, in our case dolines on the selected different lithological karstic environments in Slovenia. The solution is based on a digital elevation model (DEM) datasets with a resolution of 1, 5 and 12.5 m, applying a one-dimensional Fast Fourier transform (FFT). The Fast Fourier transform was applied on a number of the star form profiles that were positioned to the 11 selected areas with different geological settings. Since the cyclicity/periodicity of the Karst surface in Slovenia was already confirmed manually in the 1980s, we confirmed it using digital data, corresponding algorithms of the Fourier analysis, on the smoothed and normalized profiles. Since the main problem is preprocessing datasets and interpretation of the results, further solutions in this direction are proposed, such as employing other methods for normalization of the digital elevation model.

1 Introduction

Cyclicity is under certain limits ubiquitous and present in many natural and other phenomena, starting with the in Earth rotation and Milankovitch cycles. In our study, we focus on the analysis of relief cyclicity for the case study of karst surface in Slovenia. Already in the 80s, Šušteršič (1985) used the Fourier analysis for karst geomorphology in Slovenia and found out some periodicity with a wavelength of approximately 390 m and another at 150 m, which he has attributed to tectonic influence.

The usage of surface cyclicity is relatively rare. Some latest examples are detecting the landforms in a combination of Fast Fourier transform (FFT) and singular value decomposition (Wieland and Dalchow, 2009), detection of geological delineations (Mugglestone and Renshaw, 1998), quantifying changes in vegetation (Couteron, 2002), study of characteristic spatial scales (Perron et al., 2008), crater studies (Morita et al., 2010), and estimation of surface errors (Toth et al., 2014).

Several approaches have been used for analysis of the karst relief, especially for the tropical and cockpit karst. Harrison and Lo (1996) have used Fourier analysis for the study of tropical karst in Puerto Rico, while Fleurant et al. (2008) and Lyew-Ayee et al. (2007) for cockpit karst in Jamaica. Ćalić (2009) has used this approach to study the formation and characteristics of uvalas (compound enclosed karst depressions; Ford and Williams, 2007). Yet, no comparison was done to study the cycles in several different lithological karstic environments, what is another significant motivation for this study.

Even rarer is the use of Fast Fourier transform (FFT) for analysis of karstic features. Karst covers large areas of the Earth surface, and carbonates are known to be among the most complicated rocks to analyze due to its heterogeneity. Such difficulties are known from hydrogeological aspects (Neuman, 2005), structural analyses, as well as in geomorphology. Apart from the absence of surface water in karst, the most obvious distinction between the karstic and common fluvial surface is the nature of surface formation. Karst surface is namely a reflection of underground karstic features and so influenced by rock “underground”, with the most important influencing factor in the karst permeability formation being the faulting and fracturing of the carbonate rocks. These produce fractures, which can later evolve in karst channels and shafts. Several types of geomorphological features can be observed on the surface (Ford and Williams, 2007), and all can influence the cyclicity of the surface. By far most important are dolines, with usual diameters from 10 to more than 100 m and depths to approximately 10-20 m. These are formed by the vertical outflow of rainwater through the fractures and shafts in the lowest parts of dolines.

The fast Fourier transform (FFT) is an algorithm based on a Discrete Fourier Transform (DFT). It is a well-known mathematical discrete transformation of the time domain to the frequency domain, with the intention to extract the elementary sinusoidal cycles from a complex sum of wavelengths. It is a widely used method for signal processing in electrical engineering, and due to its capacity to extract cycles, it has also found its use in geology in several different fields, with most uses in time-related sedimentological cycles and geomorphological analysis. The latter was already performed in the 60's and 70's (Rayner, 1972; Pike and Rozema, 1975; Hanley, 1977).

The aim of this paper is to use an automated approach based on a digital elevation model (DEM) and applying the Fast Fourier transform (FFT) to dolines in a regional scale, on the selected different lithological karstic environments in Slovenia.

2 Materials and Methods

2.1 Study area

We have selected 11 different geological settings for the study in Slovenia (Europe), with the central coordinates of the capital Ljubljana 46°05' N, 14°50' E, inside the following carbonate outcrops (Fig. 1): Suha krajina (Dry Carniola) (1), Ravnik area (2), Trnovski gozd (3), Kras (Classical Karst) (4), Podgorski kras (5), Matarsko podolje (6), Snežnik (7), Jelovica (8), including three non-karstic areas of Goričko (9), Polhov Gradec (10), and Primorje (Coastal area) (11)).

2.2 Datasets

The datasets, a DEM12.5 (12.5 m resolution grid) was designed with data conflation methods (Podobnikar, 2005), a DEM5 (5 m resolution grid) is photogrammetrically enhanced earlier DEM, while a DEM1 (1 m resolution grid) is the newest Lidar DEM. The DEM12.5 and DEM5 were used for all the 11 study areas, while the Lidar DEM1 was used only for the Kras area (4). The datasets were provided by Surveying and Mapping Authority of the Republic of Slovenia.

2.3 Star form profiles design

The first part of the methodological design is star forms design. Cyclicity was designed to quantify the results on the profiles, placed to target points on the DEM, in four different directions (N–S, NE–SW, E–W and SE–NW). In that way, the four profiles form, actually an eight-spoked asterisk structure (✳) is later in this paper called "star form profiles". The eight-spoked star is conceptually slightly similar to the concepts of well-known D8 algorithms in GIS or DEM analysis, and even to the radar/star chart. We have used the 1-D FFT instead of 2-D, as it was possible to orient the profiles in several different directions, to simply study the anisotropy effects and consequently the influence of tectonic structures.

We arbitrary determined and set the length of the profiles to 2000 m taking into account for variations of the surface and for influences of the morphological features. Profiles were limited to this length, as long they can generally include geologically and morphologically dissimilar regions. Furthermore, centers of such star-shaped profiles were placed in interval of 200 m (i.e. 200 m apart, Fig. 2). On the other hand, this distance was chosen in such a way that profiles could remain in the same lithology. In each individual area, at least 35-star forms were placed on the DEM.

Figure 1: Locations of the 11 study areas, presented with star form profiles indicated by different colors.

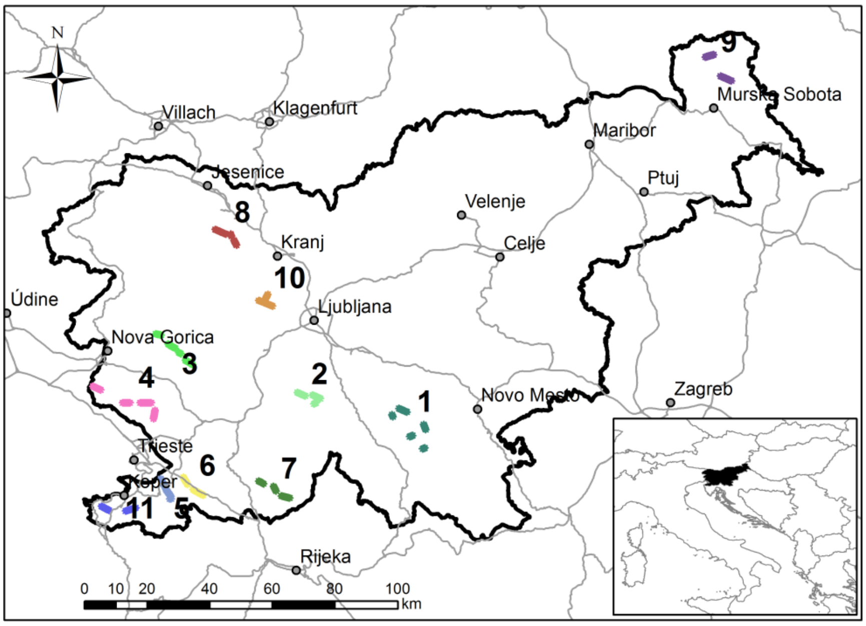
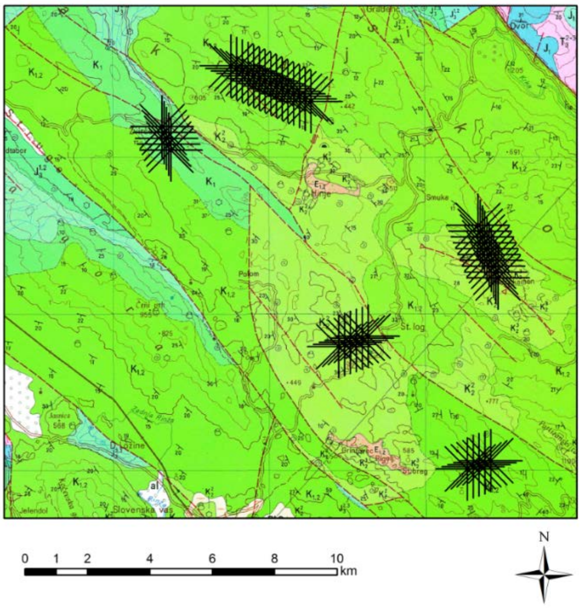


Figure 2: The area 1 (Suha krajina) with the star form profiles in Cretaceous limestones. The geological maps in the background is after (Pleničar et al., 1965).



The next step is to extract the surface of elevations from the DEM profiles and save these elevations as numerical values to the database form. The elevations were recorded considering the original resolution of the DEMs.

2.4 Fourier analysis

The second part of the methodological design is to develop the Fourier analysis, more particularly, a fast Fourier transform (FFT). Our goal here is to reveal whether cyclicality occurs on the karst (and non-karst surface), where we do not handle with time-dependent data, but with spatially changing data. Therefore, we replace the time variable to the distance (the star form profiles). Using the star form profiles, we are going to find such function or a smaller number of functions that best describe the morphology of the target surface.

In mathematical terminology, the form of the profile is called "wave" (Cartwright, 1990), while in electro engineering terminology is called "signal" (Peters, 2017). Hereinafter we will use a term signal.

The Fourier analysis is a mathematical concept used in the information sciences. This method allows decomposition of a non-sinusoidal (periodic) signal to the sum of pure trigonometrical functions, such as sine and cosines (Cartwright, 1990). The method for decomposition of the function on more basic parameters was first introduced by a French physicist and mathematician Jean-Baptiste Joseph Fourier (1768-1830). Fourier developed only Fourier's series that represent a periodic function or a periodic signal as the sum of an infinite number of basic sinusoidal and cosine functions that differ in amplitude, frequency, and phase. The interval is finite but it repeats itself to infinity.

The complex signal can be decomposed to several basic sinusoids by Fourier analysis, i.e. by Fourier Transform (FT). The FT converts the continuous signal from the time domain into a frequency domain. Similarly is possible to transform discrete values – points using the Discrete Fourier Transform (DFT). The DFT also allows the reassemble of the signal from unbundled sinusoids. This operation is called an inverse Discrete Fourier Transform (IDFT). While the DFT can be applied to any complex valued series, for large series, it can take much time to compute Fourier coefficients. Cooley and Tukey (1965) developed a much faster algorithm called Fast Fourier Transform (FFT). Nowadays there are many FFT algorithms based on a range of theories. In practice, we implemented an HFT algorithm. The result of the HFT is Fourier coefficients, which are complex numbers. The real part of them represents the amplitude, while the imaginary part presents the amplitude and phase of the signal.

Before applying the FFT, in this study, the surface of elevations from the profiles was preprocessed with truncation of the profile values to remove high-frequency noise, and normalized allowing comparison with trend elimination. From the original values was therefore eliminated the trend, i.e. simply by subtracting the mean value of the profiles (Priestas, 2010). The average value of every normalized profile was consequently set to 0. This step allows for a later comparison of the power of the different profiles in the spectral plots with periodograms (one of the time-series plots). In addition, all cycles were ranked by their periodogram values, to find out the strongest cycles.

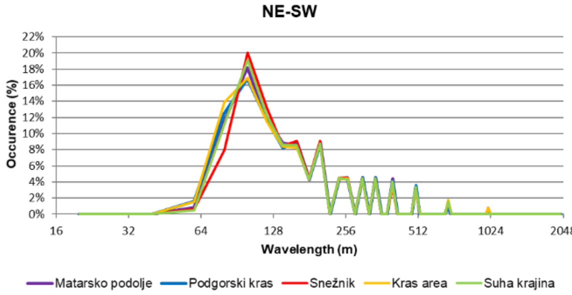
The wavelengths of the profiles were later on presented with histograms with 20 m interval bins, presenting the maximum value in the each class, all together 100 classes. We didn't include all wavelengths in the graphs because many of those with small values, in our case less than 50 m, present the noise. Therefore, in the histograms are included the 22 largest wavelengths arranged by the amplitude power.

3 Results and discussion

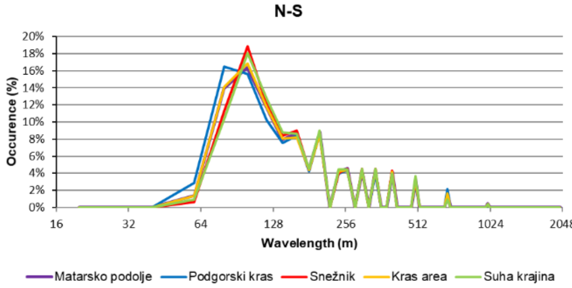
3.1 Comparison between different directions

Comparison of relative frequencies (wavelengths with most occurrences) was at the beginning performed for Cretaceous limestones on the following areas: Matarsko podolje (6), Podgorski kras (5), Snežnik (7), Kras area (Classical Karst, 4) and Suha krajina (1). Fig. 3 shows the periodograms with the wavelengths from 0 to 2000 m and relative occurrence in percentage. In all areas, most wavelengths appear with a dominant peak at around 80 to 100 m where the greater part lies in the range from 60 m to about 200 m. This somewhat corresponds to the wavelengths around 155 m obtained by (Šušteršič, 1985), but not with the ones of 390 m. Some minor peaks occur at longer wavelengths, but the longest ones should be disregarded, as they occur at higher frequencies of the longest 2000 m interval length. Therefore, the wavelength of 2000 m should be considered meaningless, and the same conclusion can be made for its multiples (1000 m, 500 m, etc.). In addition, there is no difference between the five study areas with the same lithology. Some minor differences appear at shorter wavelengths up to 100 m but are insignificant.

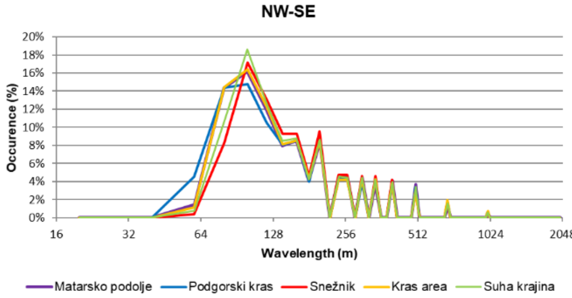
Figure 3: Periodograms: percentage of wavelengths – Cretaceous limestones, four directions.



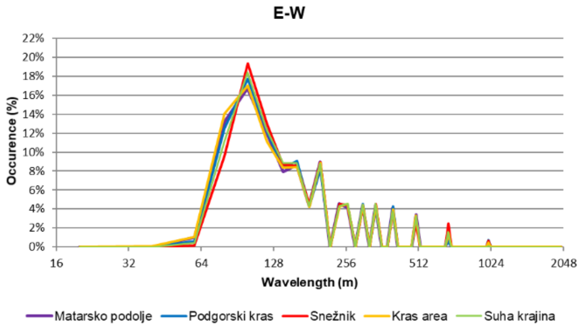
(a)



(b)



(c)



(d)

Regarding the dependence of wavelengths from the profile directions, small but still detectable differences can be seen in Fig. 4 for the direction NW-SE, which corresponds to the direction of “Dinaric” faults.

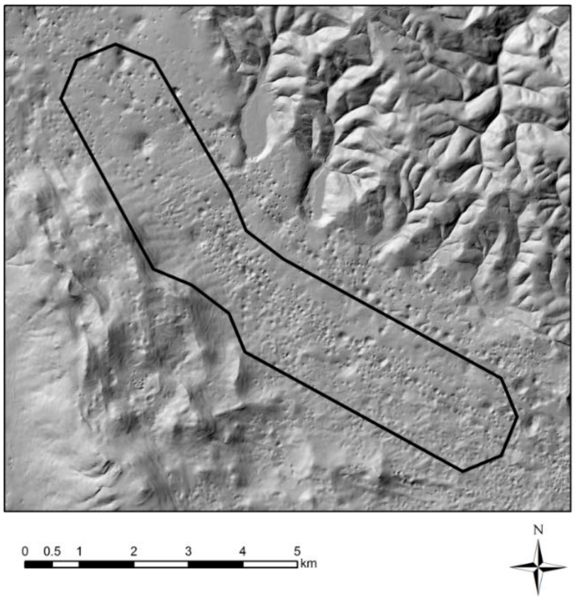
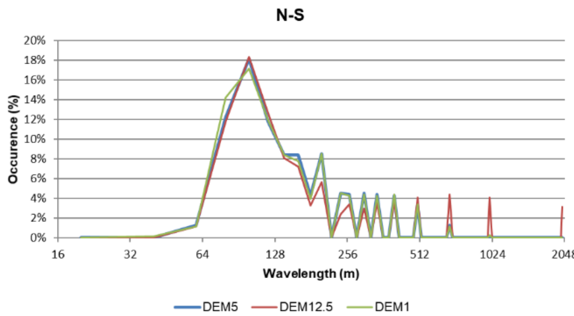


Figure 4: Study area of Matarsko podolje (2) (hillshaded DEM5).

3.2 Comparison between different DEMs

With using the 1 m resolution Lidar DEM it is possible to detect smaller variations (wavelengths) in relief compared to the coarser DEMs with resolutions of 5 and 12.5 m. These variations represent smaller dolines, which are not mapped on the coarser DEMs and even a noise. However, the differences are negligible. Only a periodogram for the direction (N-S) is presented here, as the same results were obtained for the other directions (Fig. 5).

Figure 5. Periodogram with the percentage of wavelengths according to different DEMs in direction N-S.



4 Conclusions

The 1-D Fast Fourier transform method was applied to the star form profiles on the bare surface. The study shows different results only for smaller wavelengths when using digital elevation models (DEM) in a resolution of 1, 5 and 12.5 m. Small but still detectable differences were observed for the direction NW-SE with regard to the other directions, which corresponds to the direction of Dinaric faults.

As part of the applied geographical information science, the proposed approach could be implemented to any other type of landform in various geographic scales (or resolution), for example to the Alpine surface, to study cyclicity of the particular terrain. To improve the cyclicity assessment of the karst dolines, we propose the following two solutions for further study: (A) using the Fast Fourier transform method for the analysis in more mature karstic terrain, and (B) improve normalization of the profiles with a non-linear adaptive trend (Tian et al., 2018).

The proposed geospatial analysis applies Fourier analysis to the GIScience that could help in understanding geomorphological formations or in development of landform taxonomy, as well as in natural disaster management like as for prediction potential of hazardous torrential fans/talus cones.

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